Photon-Mediated Peierls Transition of a 1D Gas in a Multimode Optical Cavity

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The Peierls instability toward a charge density wave is a canonical example of phonon-driven strongly correlated physics and is intimately related to topological quantum matter and exotic superconductivity. We propose a method for realizing an analogous photon-mediated Peierls transition, using a system of one-dimensional tubes of interacting Bose or Fermi atoms trapped inside a multimode confocal cavity. Pumping the cavity transversely engineers a cavity-mediated metal-to-insulator transition in the atomic system. For strongly interacting bosons in the Tonks-Girardeau limit, this transition can be understood (through fermionization) as being the Peierls instability. We extend the calculation to finite values of the interaction strength and derive analytic expressions for both the cavity field and mass gap. They display nontrivial power law dependence on the dimensionless matter-light coupling.

Introduction.—The interaction between electrons and phonons has traditionally played a leading role in the formation of quantum phases of matter, with superconductivity being a prime example. Quantum simulation in optical lattices provides an enticing platform for exploring new phases [1], but phonon-driven physics lies beyond traditional optical lattice capabilities, as they are externally imposed and rigid. The use of high-finesse optical cavities has been suggested as a route to overcome this by making the optical lattice fully dynamical and compliant [2–4]. This requires cavities that support multiple degenerate modes, as single-mode cavities only allow dynamics of the lattice intensity, not its period. That is, while single-mode cavities have provided access to a diverse array of exotic quantum phenomena including self-organization [5], supersolids [6], spinor density-wave polariton condensates [7], dynamical Mott insulators [8,9], and dynamical spin-orbit coupling [10], only multimode cavities support fully emergent optical lattices whose amplitude and periodicity may vary [2–4]. Multimode cavity experiments have already engineered a variety of photon-mediated interatomic interactions [11–14]. These could lead to the creation of new many-body systems and states of matter such as quantum liquid crystals made of photons and superfluid atoms [3,4] and superfluids exhibiting Meissner-like effects [15].

As we will show below, confocal multimode cavities coupled to one dimensional (1D) quantum gases provide a way to realize controllable electron-phonon-like interactions using ultracold atoms. Other proposals for studying this physics include coupling fermionic atoms to an optical waveguide [16], or to a crystal of trapped ions [17]. One-dimensional ultracold gases also allow one to explore pairing physics with bosons, as resonant atomic collisions provide a knob to make bosons strongly repel [18,19], even to the point that they behave like fermions [20–22]. For such systems, the addition of attractive interactions can cause dramatic effects. Indeed, for 1D systems, even weakly attractive interactions result in instabilities leading to strong correlations such that the quasiparticle picture breaks down and collective modes emerge [23–25]. A paradigmatic example is the Peierls instability that occurs because the susceptibility of a free Fermi gas diverges due to infinitesimal density perturbations at twice the Fermi wave vector [26]. If free phonons exist, then it is possible for the system to create an emergent lattice that matches this wave vector. Because of the diverging susceptibility, the system undergoes a metal-insulator transition and dynamically generates a mass gap. The Peierls transition is a canonical example of phonon-driven physics and is intimately related to the continuum Su-Schrieffer-Heeger (SSH) model of 1D topological insulators in 1D [27]. Numerical work has shown the analogue of a Peierls transition for atoms in an optical lattice, where intersite hopping of bosonic atoms is modulated by the spin state of a second species of atoms [28].

In this Letter, we show that a Peierls instability occurs in a strong, repulsively interacting 1D bosonic gas trapped inside a transversely pumped confocal multimode optical cavity. Building on demonstrated experimental capabilities [12,13], we predict that, by tuning the interatomic interactions to the hard-core, Tonks-Girardeau (TG) limit

[29,30], the cavity can mediate a Peierls transition in the bosonic gas, with a mass gap and photon amplitude that is exponential in the matter-light coupling. By using bosonization, we then extend these calculations to finite values of the interatomic interaction, as well as to interacting fermionic systems. In agreement with Ref. [28], we show that, in these cases, the cavity can mediate a metal-insulator transition, albeit one of different character. Moreover, we show that the dynamically generated mass gap and photon amplitude have a nontrivial power law dependence on the matter-light coupling. Self-organization of fermions in a single-mode cavity has also been previously discussed, theoretically [31–33], but here, the diverging susceptibility requires the single cavity mode and Fermi wave vectors to match. Experimentally, realizing a photon-mediated Peierls transition would open new avenues toward exploring the role of Fermi surface nesting and charge density waves in exotic superconductors in simulators operating in a quantum-optical, many-body context.

Model.—The system considered, depicted in Fig. 1, consists of 1D tubes of atoms placed in a transversely pumped confocal optical cavity. As already noted, multiple optical modes are needed to allow a fully emergent optical lattice. For true multimode operation, these modes must be degenerate or near degenerate. A confocal cavity is the simplest stable resonator allowing such degeneracy [34]. To achieve a 1D trap geometry and uniform atom-cavity coupling, we confine bosonic atoms in a strong λ_T-periodic 2D optical lattice formed by a retroreflected beam along the $\hat{y}$ pump direction and an intracavity standing wave along the $\hat{z}$ cavity axis. A pump field along $\hat{y}$ has a wave vector k_r close to a cavity resonance. By choosing the tube-lattice period such that $k_r\lambda_T/2\pi$ is an integer, the tubes lie at the peaks of the pump and cavity standing-wave fields so the atoms coherently Bragg scatter light into the cavity. As a result, in contrast to experiments on self-organization [35],
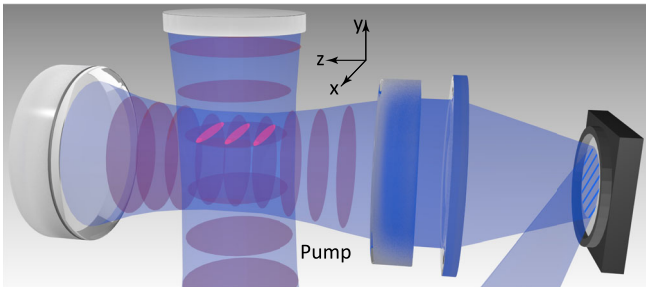

FIG. 1. Schematic of our system. A gas of interacting atoms is placed in a confocal optical cavity that supports many degenerate spatial modes. A transverse pump (blue) along $\hat{y}$ scatters photons off the atoms (red) into the multimode cavity field (blue). The gas is confined by a 2D optical lattice (purple) and offset from $y = 0$ to avoid mirror-image interactions [12]. The amplitude and phase of the light is imaged via holographic reconstruction of a spatial heterodyne signal formed by interfering part of the pump with the emission.

there is no spontaneous atomic organization in the yz plane; rather, the atoms superradiantly emit into the cavity regardless of pump strength. We choose the tubes to be near the cavity midplane $z = 0$, and the long Rayleigh range of a confocal cavity ensures that the tubes at different z will behave identically. In $\hat{y}$, we choose all tubes to be centered at the same y since tubes at different y decouple, as discussed in the Supplemental Material [36]. As such, we will describe the atoms in tube t through a bosonic field $\Psi_t(x)$, varying only along the free x direction. Degenerate confocal cavities with Bose-Einstein condensates in optical traps are practicable with existing technology [14].

We can write the Hamiltonian of the system as follows [36]:

$$H = H_{\text{cav}} + \int dx \sum_{t=1}^{N_z} \left\{ \Psi_t^\dagger(x) \left[-\frac{\hbar^2}{2m} \partial_x^2 - \mu \right] \Psi_t(x) \right.$$
$$\left. + U\Psi_t^\dagger(x)\Psi_t(x)\Psi_t^\dagger(x)\Psi_t(x) - g\Phi(x)\rho_t(x) \right\}. \quad (1)$$

The first term, $H_{\text{cav}} = \hbar \sum_{\alpha,\nu} \omega_{\alpha,\nu} a_{\alpha,\nu}^\dagger a_{\alpha,\nu}$, describes the cavity photons, where $\omega_{\alpha,\nu}$ is measured with respect to the transverse pump frequency. We sum only over cavity modes that are near resonant with the pump. Modes are labeled by the longitudinal index α and transverse index ν. The second and third terms describe atoms with mass m, chemical potential μ, and a contact interaction of strength U. We sum over an array of N_z tubes, labeled by t, positioned in an array along $\hat{z}$. The last term describes the coupling of the atomic density $\rho_t(x) = \Psi_t^\dagger(x)\Psi_t(x)$ to the cavity photons, as induced by the pump. This term describes how the atomic density scatters photons between the transverse pump and the cavity modes. As described in Ref. [36], this term can be derived by adiabatically eliminating excited states of the atoms, yielding an effective ac light shift. The photon field is written as a sum over cavity modes $\Phi(x) = \sum_{\alpha,\nu} c_\nu^\alpha \tilde{\Xi}_\nu(x)(a_{\alpha,\nu}^\dagger + a_{\alpha,\nu})$. The transverse mode functions $\tilde{\Xi}_\nu(x)$ are found by taking the eigenmodes of the cavity—Gauss-Hermite functions of order (l_ν, m_ν) in the x and y directions, respectively—and convolving these with the Gaussian tube profile in the y direction; see Ref. [36]. The factors c_ν^α come from the longitudinal spatial mode profile evaluated at the atom positions, and are discussed below. As noted above, this final term is a source for cavity photons independent of the density profile $\rho_t(x)$—the $\lambda - T$ tube spacing causes atoms to coherently scatter the pump into the cavity with intensity $\propto N_z^2$ [40]. The prefactor $g = \hbar g_0 \Omega/\Delta_a$ is the effective matter-light coupling where g_0 is the bare coupling, Ω the pump Rabi frequency, and Δ_a is the pump-atom detuning.

As the Gauss-Hermite functions form a complete basis set, one might expect that $\Phi(x)$ could take any spatial profile. This would allow the cavity light to match the

atomic density, inducing a local interaction. There are complications however. First, while transverse modes become degenerate at confocal resonances of the cavity, they do so in alternating odd and even families, set by the parity of $n_\mu = l_\mu + m_\mu$. We assume even n_μ hereon. Second, the factors c_ν^α modify the sum over modes. As shown in Ref. [13], the longitudinal mode profile assumes a form $c_\nu^\alpha = \cos(\xi^\alpha - \pi n_\nu/4)$, where ξ^α depends on which family we consider. This includes the effects of the Gouy phase [34], leading to an n_ν dependence. If we consider the special case where ξ^α is a multiple of 2π, we then see that for successive even n_μ, the factor c_ν^α is sequentially $1, 0, -1, 0$. The missing Gauss-Hermite functions prevent $\Phi(x)$ from obtaining an arbitrary form; equivalently, this yields a nonlocal photon-mediated interaction. This can be nulled by using two pumps resonant with families $\xi^\alpha = 0$ and $\pi/2$, yielding a local interaction [13].

Both the matter-light coupling g and interatomic interaction strength U are experimentally tunable parameters [1]. In particular, the system may be tuned into the TG regime, i.e., $\gamma \equiv mU/\hbar^2\rho_0 \to \infty$, using tight trapping and collisional resonances, where ρ_0 is the average 1D density [20,21]. The atoms behave like free fermions in this limit [29,30] and, so, will exhibit a Peierls instability. Even strongly repulsive bosons away from the TG limit exhibit this instability.

Steady state.—We investigate the Peierls instability using a mean-field description of the photon field. Therefore, we consider the equations of motion for the expectation of photon operators

$$\langle \dot{a}_{\alpha,\nu}^\dagger \rangle = (i\omega_{\alpha,\nu} - \kappa)\langle a_{\alpha,\nu}^\dagger \rangle - i\frac{g}{\hbar}\sum_t \int dx\, c_\nu^\alpha \tilde{\Xi}_\nu(x)\langle \rho_t(x) \rangle,$$
(2)

where we have included a term $\propto \kappa$ accounting for cavity losses. Assuming a steady state, the mean field is

$$\langle \Phi(x) \rangle = \int dx' \sum_{\alpha,\nu,t} \frac{2g\omega_{\alpha,\nu}(c_\nu^\alpha)^2}{\hbar(\omega_{\alpha,\nu}^2 + \kappa^2)} \tilde{\Xi}_\nu(x)\tilde{\Xi}_\nu(x')\langle \rho_t(x') \rangle. \quad (3)$$

For simplicity, we consider the perfectly degenerate limit $\omega_{\alpha,\nu} = \omega$. Pumping two families as discussed above, $\sum_\alpha (c_\nu^\alpha)^2 = 1$, allowing for the explicit evaluation of the sum over modes: $\sum_\nu \tilde{\Xi}_\nu(x)\tilde{\Xi}_\nu(x') = [w_0^2/(2\sqrt{4\pi}\sigma_T)]\delta(x - x')$ [41]. Here, σ_T is the transverse width of an individual tube, w_0 is the beam waist, and we made a simplifying assumption that the tubes are in the upper half of the cavity, $y > 0$, to avoid mirror-image interactions [12]. As all tubes behave identically, we can insert this into Eq. (3) to give

$$g\langle \Phi(x) \rangle = \pi\hbar v_F \eta \langle \rho(x) \rangle, \quad \eta \equiv \frac{g^2 N_z \omega w_0^2}{\sqrt{4\pi}\sigma_T \pi\hbar^2 v_F(\omega^2 + \kappa^2)},$$
(4)

where we have defined the dimensionless matter-light coupling η and $v_F = \pi\hbar\rho_0/m$ is the Fermi velocity in the TG limit. Later, it will be convenient to parametrize this as $\langle \Phi(x) \rangle \equiv \Phi_0(x) + \Phi_{2\pi\rho_0}(x)[e^{2i\pi\rho_0 x} + e^{-2i\pi\rho_0 x}]$ and, also, introduce the quantity $\Delta \equiv |g\Phi_{2\pi\rho_0}|$. The atoms will coherently scatter light into the cavity, so any nonzero density of atoms implies $\langle \Phi \rangle \neq 0$. We will find that $\Phi_{2\pi\rho_0}(x)$ becomes nonzero at the Peierls transition, leading to the dynamical generation of a mass gap.

Adiabatic elimination of photons from our model at the mean-field level leads to completely conservative dynamics—this is a generic feature of a Rabi-like matter-light coupling [42]. The resulting conservative dynamics is determined entirely by an effective Hamiltonian, H^{eff}, and so, the steady-state condition becomes equivalent to minimizing this with respect to the mean field Δ. The atomic and atom-cavity coupling parts of H^{eff} come from substituting Eq. (4) into Eq. (1) (considering a single tube), while the cavity part can be written as $H_{\text{cav}}^{\text{eff}} = (g^2/2\pi\hbar v_F \eta) \int dx \langle \Phi(x) \rangle^2$. We consider only constant values of Φ_0 and $\Phi_{2\pi\rho_0}$, in which case Φ_0 can be absorbed into a redefinition of the chemical potential, μ. Henceforth, we shall deal only with $\Phi_{2\pi\rho_0}$.

Low energy and bosonization.—The atomic system can be described using bosonization, which provides us with a description of our system in terms of two new bosonic fields, $\phi(x)$ and its canonical conjugate $\partial_x\theta(x)$ [23,24,43,44]. The former is related to the atomic density via $\rho(x) = [\rho_0 - (1/\pi)\partial_x\phi(x)]\sum_{n=-\infty}^\infty e^{2in[\pi\rho_0 - \phi(x)]}$, while the latter is related to the current in the system [44]. In terms of these, the steady-state condition Eq. (4) reduces to $g\Phi_{2\pi\rho_0} = 2\pi\hbar v_F \eta\rho_0\langle\cos[2\phi(x)]\rangle$. As such, the effective mean-field Hamiltonian discussed above can be written in terms of the bosonized fields as

$$H^{\text{eff}} = H_{\text{cav}}^{\text{eff}} + \frac{\hbar v_F}{2\pi}\int dx \left\{\frac{1}{K^2}[\partial_x\phi(x)]^2 + [\partial_x\theta(x)]^2\right\}$$
$$\pm 2\Delta \int dx \rho_0 \cos[2\phi(x)]. \quad (5)$$

The first line describes the atomic and cavity systems, using the standard result for bosonization of the atoms. The atomic interactions are encoded via the parameter K which depends on U in a complicated fashion [45]. For large repulsive interactions, this relationship can be approximated by $K \approx 1 + 4/\gamma$, with the TG limit achieved at $K = 1$, while $K = \infty$ corresponds to free bosons [45]. The second line describes the relevant part of the matter-light coupling and will generate a gap. We have allowed for the possibility that the matter-light coupling and photon field might carry opposite signs, and we keep only terms which are most relevant in a renormalization group sense, which restricts our analysis to values $1/2 < K < 2$.

While, in the present work, our primary system of interest is bosons with short-range, repulsive interactions, bosonization also allows one to describe the low-energy physics of bosons with long-range interactions or interacting fermions [23,24,46]. Such systems are described by a Luttinger parameter $0 < K < 1$, and so, in the following, we allow for arbitrary values of $1/2 < K < 2$. Results for $K > 1$ are applicable to bosons with short-ranged interactions or fermions with attractive interactions, whereas $K < 1$ corresponds to 1D repulsive fermions or bosonic systems with long-range interactions, the latter of which has been realized in Refs. [47,48].

Tonks-Girardeau limit.—The atoms behave as free fermions in the TG limit [29,30]. This is evident in our low-energy description at $K = 1$, where it is possible to express the bosonic operators as a pair of chiral fermions. In terms of these, the low-energy Hamiltonian Eq. (5) becomes the 1D Dirac Hamiltonian with a mass $\pm\Delta$ [36,49] that is subject to the steady state condition. This system is the same as the SSH model [27] whose solution is well known. Carrying it over to the present case, we find that $\Delta = 2E_F e^{-1/\eta}$, where $E_F = \pi\hbar v_F\rho_0$ is the Fermi energy. Therefore, the self-consistent photon field is given by

$$\langle\Phi(x)\rangle = \pm\frac{4E_F}{g}e^{-1/\eta}\cos(2k_F x). \quad (6)$$

The atomic system is insulating with a mass gap of 2Δ. The applicability of the low-energy description relies on E_F being the largest scale in the system. In particular, we require that $\Delta < E_F$, which in turn requires $\eta < 1$.

Finite interaction.—The system is quite different for $K \neq 1$. It is strongly correlated and interacting, but can no longer be mapped to the SSH model. Nevertheless, an exact solution for the steady state can be found, although the cases of positive and negative $g\Phi_{2\pi\rho_0}$ need to be treated separately; below, we will find that these correspond to $K > 1$ and $K < 1$, respectively. For $g\Phi_{2\pi\rho_0} < 0$, the atomic part of the Hamiltonian is that of the Sine-Gordon or massive Thirring model with a positive mass term [23,24,43]. This is an exactly solvable field theory, and many of its properties are well known [50,51]. In particular, the mass gap of the model becomes renormalized due to the interactions, $\Delta_R^+ = \xi_+ E_F[\Delta/E_F]^{1/(2-K)}$, with ξ_+ a K-dependent constant provided in Ref. [36]. Using this, we derive the following steady-state condition from which to determine $\Phi_{2\pi\rho_0}$:

$$\frac{\Delta}{E_F} = -\eta\frac{\pi\xi_+^2}{2}\cot\left(\frac{\pi}{2-K}\right)\left[\frac{\Delta}{E_F}\right]^{\frac{K}{2-K}}. \quad (7)$$

This has a solution only for $K < 1$. Rearranging, we find that the self-consistent photon field and mass gap are

$$\langle\Phi(x)\rangle = -\frac{2E_F}{g}\zeta_+\eta^{\frac{2-K}{2-2K}}\cos(2\pi\rho_0 x), \quad (8)$$

$$\Delta_R^+ = \xi_+ E_F[\zeta_+\eta]^{\frac{1}{2-2K}}, \quad (9)$$

where ζ_+ is given in Ref. [36]. The amplitude of the photon field now has a power law rather than the exponential dependence on η in the TG limit. Furthermore, the exponent differs from that appearing in the mass gap. The TG limit $K \to 1$ cannot be recovered from the above expression and must be treated separately as in the previous section. This highlights the strong correlations in the interacting system. The photon field oscillates at wave vector $2\pi\rho_0$, which is $2K$ times k_F.

The model maps to the massive Thirring model for $g\Phi_{2\pi\rho_0} > 0$, but with a negative mass parameter. As explained in Ref. [36], this change in sign of the mass term results in the spectrum of the model being inverted; i.e., the ground state becomes the highest excited state [36]. Spectral inversion also occurs when changing the sign of the interactions—i.e., taking $K \to 1/K$. Combining these, we find that for $g\Phi_{2\pi\rho_0} > 0$ and $K > 1$, the renormalized mass gap and self-consistent photon field are

$$\langle\Phi(x)\rangle = \frac{2E_F}{g}\zeta_-\eta^{\frac{2K-1}{2K-2}}\cos(2\pi\rho_0 x), \quad (10)$$

$$\Delta_R^- = E_F\xi_-[\zeta_-\eta]^{\frac{K}{2K-2}}, \quad (11)$$

where ξ_-, ζ_- are related to ξ_+, ζ_+ by $K \to 1/K$.

Experimental signatures.—One may image the atomic density profile by using the spatial resolving power of the degenerate cavity. This provides a direct signature of the Peierls instability as a density wave. Figure 2 shows this, calculated using results from Ref. [13]. The spatial modulation of the light amplitude from the atomic image is a signature of the Peierls transition; a mirror image appears at $-y_t$. The geometry of the confocal cavity conveniently provides both the emission of the atomic density image (long thin tubes) and its Fourier transform (vertical stripes) [13]. Thus, a density modulation of wave vector $2k$ results in Bragg peaks manifest as vertical stripes. These are positioned at $x = \pm k w_0^2$. (Each stripe is the Fourier transform of the $\hat{y}$-displaced atom image.)

The power-law scaling between the oscillation amplitude of the detected light and the parameter η depends critically on the atom interactions; see Eq. (8). Thus, the Luttinger parameter K can be experimentally measured through the dependence of this exponent on η. This can be tuned through its dependence on the pump intensity $\eta \propto g^2 \propto \Omega^2$ or by the cavity-pump detuning ω.

Probing the system by stimulating a particular photon mode realizes cavity-enhanced Bragg spectroscopy [52]. In a degenerate cavity, the probe field profile can be tailored with holographic beam shaping to have a particular wave
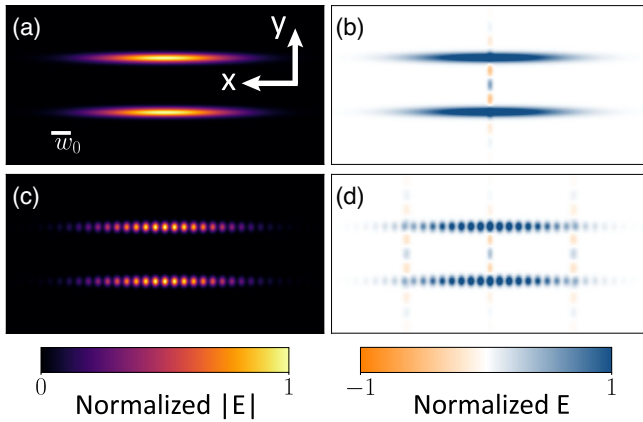
FIG. 2. Left: simulated intracavity field amplitude at the atoms (a) below and (c) far above the transition, with a density wave of wave vector $k = \pi/(0.6w_0)$ in the TG limit. Right: cavity emission (b) below and (d) above the transition. Emission from a confocal cavity contains both an image of the atomic density (horizontal blue pattern) and its Fourier transform (vertical blue and orange patterns). These can be measured by holographic reconstruction of a spatial heterodyne image [7,14]. The scale bar in (a) shows the Gaussian waist w_0 of the cavity.

vector. Thus, dynamic susceptibility can be measured as a function of k and excitation energy by also tuning the detuning between the probe and pump. The response of the system manifests as an increase in photon population, allowing the gap $\Delta_R^\pm$ to be measured.

In conclusion, we have shown that multimode confocal cavities can be used to realize the Peierls transition for both Bose and Fermi gases. Away from the simple limits of noninteracting fermions or TG bosons, the scaling of the detected light field with pump strength can be used to measure the Luttinger parameter. Looking beyond Peierls transitions, the compliant, phonon-supporting optical lattices inherent in multimode cavity QED make accessible a wider variety of many-body physics explorable in the context of quantum simulation.

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

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